

# Positive and Negative Deviations from Additivity in Drag Reduction of Binary Dilute Polymer Solutions

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It is well known from the thermodynamics of solutions that the pressure of a binary mixture is not always a linear combination of the vapor pressures of the pure components. Positive and negative deviations from Raoult's law are generally encountered. By analogy we have asked if positive or negative deviations from additivity can also occur in the drag reduction of binary polymer solutions. Turbulent pipe flow drag reduction caused by polyethylene oxide, polyacrylamide, and carboxymethylcellulose has been therefore measured in single and binary dilute solutions of polymers with concentrations in the range of 10 to 80 wppm. It has been observed that depending upon the nature of the polymers, the percentage drag reduction in the binary polymer solutions may be larger or smaller than the sum of individual effects caused by the single polymer solutions. The percentage drag reduction is defined as

$$\left( \frac{f_p - f}{f_p} \right) \times 100.$$

Some data are available, for Reynolds numbers smaller than 12,000 (White and McEligot, 1970), which show that binary solutions of two polyox with molecular weights of  $2 \times 10^5$  and  $4 \times 10^6$  have a roughly additive effect (ideal behavior) concerning drag reduction.

## EXPERIMENT

The experiments were performed with tap water at 25°C, in a copper pipe of 2.5-cm I.D. which was a part of a loop consisting of a tank, a Moyno pump driven by a variable speed motor, and a Foxboro magnetic flow meter. The pressure taps were located along the pipe at distances of 100 and 146 diameters from the upstream end of the copper pipe. The differential pressure between the two taps was measured by a transducer with a 0.67 atm. diaphragm connected to a Whittaker transducer indicator. The pressure difference was recorded on a Clevite recorder.

The following polymers were used in the experiments: polyethylene oxide (polyox WSR301) of average molecular weight  $2.5 \times 10^6$ ; polyacrylamide (Separan AP30) of average molecular weight  $1 \times 10^6$ ; and sodium carboxymethylcellulose (CMC) of average molecular weight  $2.5 \times 10^5$ . The molecular weight estimates are those indicated by the manufacturers. They have been also computed from the measured intrinsic viscosities; the obtained values are within 20% of those given above.

To prevent polymer degradation, special care was taken in the preparation of the solutions. Each run lasted only 30 sec. After testing, the polymer solution was discarded and the entire system flushed with water for 20 min. before beginning the next run. Each data point is the average of at least four points with a deviation of less than 1%.

## RESULTS AND DISCUSSIONS

Figure 1 shows the friction factor-solvent Reynolds number behavior of the binary polymer solutions while Figure 2 for single polymer solutions; more detailed data are avail-

able (Dingilian, 1972). One may conclude that the percentage drag reduction caused by the binary mixture is not the sum of the effects of the individual polymers taken

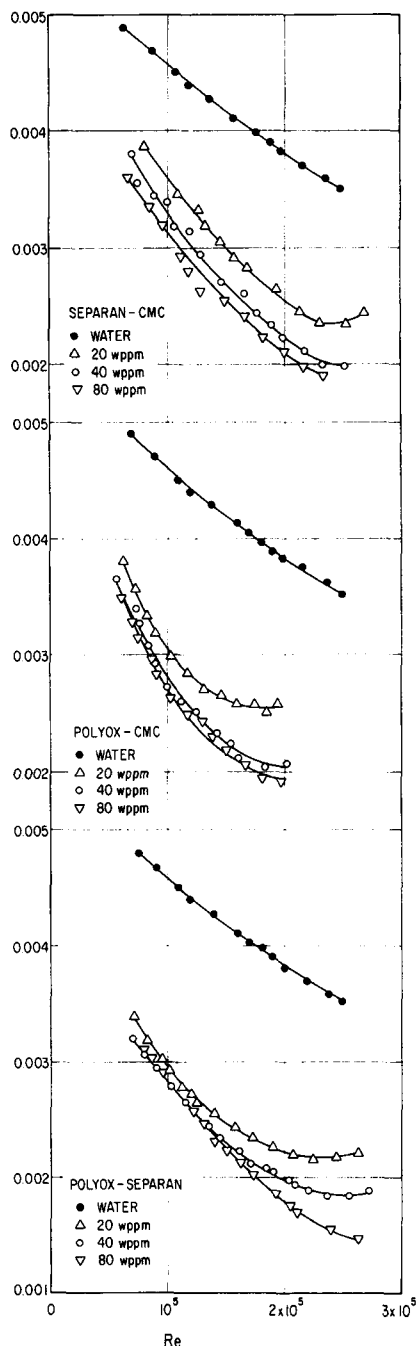


Fig. 1. Friction factor—solvent Reynolds number results of Separan—CMC, Polyox—CMC and Polyox—Separan mixtures (all 1:1 ratio).

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TABLE 1. PERCENTAGE DRAG REDUCTION AT  $Re = 200,000$  FOR SINGLE AND BINARY POLYMER SOLUTIONS

WPPM	Polyox WSR-301	Separan AP-30	CMC	Polyox-Separan (1:1)	Polyox-CMC (1:1)	Separan-CMC (1:1)
10	32.9	21.6	—	—	—	—
20	36.3	25.0	2.6	42.9	—	32.1
40	41.6	35.5	5.3	49.7	47.3	41.3
80	47.4	43.4	8.7	54.2	50.5	44.7

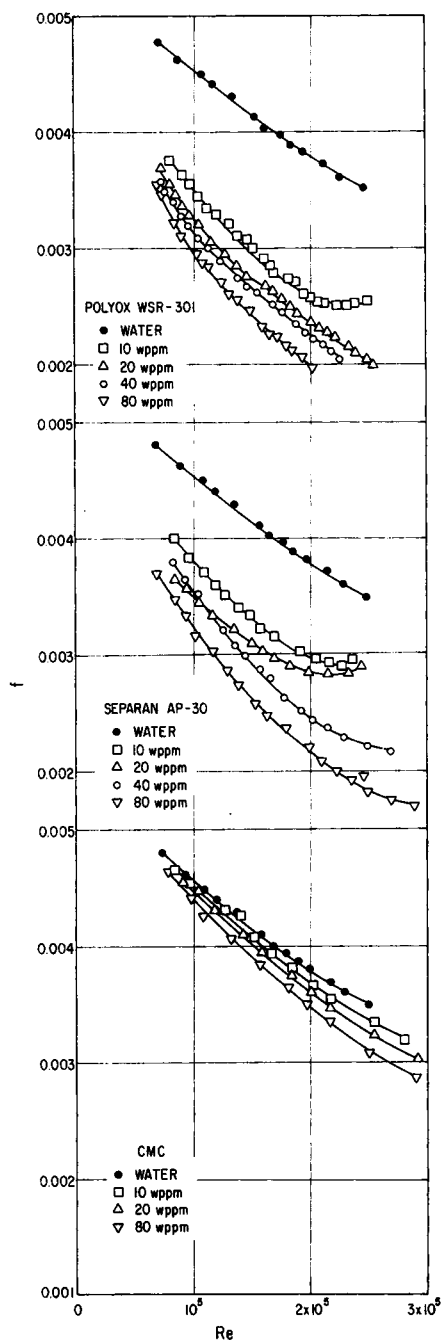


Fig. 2. Friction factor—solvent Reynolds number relationships of Polyox WSR 301, Separan AP30 and CMC.

separately. As examples, the percentage drag reduction at  $Re = 200,000$  for various systems are given in Table 1.

In the cases of Polyox-CMC and Separan-CMC systems, the percentage drag reduction of the binary polymer systems is higher (positive deviation) than the sum of the

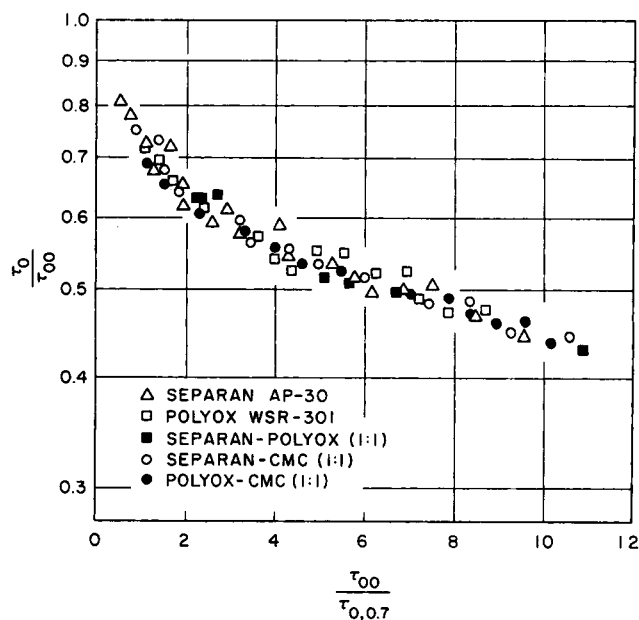


Fig. 3. The correlation of the ratio  $\tau_0/\tau_{00}$  versus  $\tau_{00}/\tau_{0,0.7}$  for sample data from these experiments.

percentage drag reductions caused by each polymer when in solution alone at the same concentration as in the mixture. The contrary observation can be made for the mixture Polyox-Separan (negative deviation). It is of interest to observe that the binary polymer solutions composed of two high molecular weight, linear, and flexible molecules as Polyox and Separan (Separan is less flexible) produced negative deviations from additivity, while the combination of a flexible molecule with the relatively inflexible polymer molecule of CMC caused positive deviations from additivity. These findings suggest that a proper choice of a binary or multicomponent polymer solution instead of a single polymer solution can significantly increase the percent drag reduction.

As a final observation we show in Figure 3 that the shear stress for the single and binary polymer solutions can be superposed on a single plot similar to that suggested by Astarita et al. (1969). The viscoelastic properties of the polymer solutions are accounted for indirectly via the shear stress  $\tau_0 = \tau_{0,0.7}$  for which  $\tau_0/\tau_{00} = 0.7$ .

#### NOTATION

- $f$  = friction factor,  $2\tau_0/\rho u^2$
- $f_p$  = friction factor for the pure solvent
- $Re$  = Reynolds number
- $u$  = average velocity
- $\tau_0$  = shear stress at the wall for the polymer solution
- $\tau_{00}$  = shear stress at the wall for the pure solvent, for the same pipe and Reynolds number as  $\tau_0$
- $\tau_{0,0.7}$  = shear stress at the wall for the polymer solution when  $\tau_0/\tau_{00} = 0.7$
- $\rho$  = density of the fluid

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Manuscript received May 13, 1974; revision received and accepted August 9, 1974.

# Characteristics of Jet Mixers—Effect of Number of Jets and Reynolds Numbers

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If a passive tracer is added to some of the jets in a jet mixer, the root mean square concentration fluctuations of the tracer can be used to characterize the turbulent mixing.

While studying the influence of turbulent mixing on chemical reactions, data have been accumulated on five mixing devices; Device I, a 1.80-mm I.D. tube located concentrically in a 3.175-mm I.D. pipe; Device II, 14 tubes, 0.394-mm I.D., close packed in a 3.175-mm I.D. pipe; Device III, geometrically similar to Device II, 14 tubes, 0.84-mm I.D., close packed in a 6.76-mm I.D. pipe; Device IV, 188 tubes, 1.37-mm I.D., close packed in a 3.175-cm I.D. pipe; Device V, 100 tubes, 1.32-mm I.D., in a 3.175-cm I.D. pipe (Vassilatos and Toor, 1965; Shuck, 1971; Mao and Toor, 1971; Singh, 1973; Toor and Singh, 1973). Very rapid acid-base reactions were used to measure  $d$  by feeding the reactants in alternate jets. This measured value of  $d$  is equivalent to what would be obtained if a tracer were fed through alternate jets (Toor, 1962; Keeler, Petersen, and Prausnitz, 1965; Vassilatos and Toor, 1965; Toor, 1969). All the data are for equal flow rates of reactant streams, equivalent to equal flow rates of tracer and nontracer fluid. The data are used in this report to examine the effect of geometry and Reynolds Number on  $d$ .

## PRELIMINARY CONSIDERATIONS

The mixing depends on the nature of the turbulent field downstream of the mixing devices and, except perhaps for Device I which is clearly a special case, the turbulence is generated primarily by the expanding, interacting jets. Since the turbulent field far downstream must decay to that field characteristic of fully developed pipe flow, one would expect that there is a region near the inlet where the mixing depends upon jet turbulence and one far downstream where the mixing depends upon pipe turbulence.

However, the observable range of  $d^2$  in the experiments is order 1 to  $10^{-3}$  and in all cases the lower bound is reached in less than 3 pipe diameters. This distance is small compared to that required to establish fully developed pipe flow, which suggests that over the region of interest the jet turbulence would control. Hence, in comparing mixing devices which are not geometrically similar, one would expect the appropriate characteristic length to be jet diameter.

Furthermore, when comparing Device II (or III which is geometrically similar to II) with Device IV, the above arguments would suggest that both devices would be identical when scaled in terms of jet diameter since the pipe wall is presumably playing no role. Looked at another way, with enough jets present most of the jets should be unaware of the wall and the central portion of a hypothetical Device II' which has the same diameter jets as Device IV but 14 jets (in a smaller pipe) should have the same central field as Device IV. But Device II' is geometrically similar to Device II so that when scaled by jet diameter the central portions of Devices II and IV should be similar.

Since Device V does not have close packed tubes the above arguments do not necessarily hold.

## COMPARISON OF MIXING DEVICES

Data for Devices II, III, and IV from various investigators (Mao and Toor, 1971; Shuck, 1971; Singh, 1973) at almost equal values of jet Reynolds number are plotted against dimensionless length in Figure 1. The data can be represented by a single curve, which confirms the earlier arguments that with a sufficient number of jets in the same array jet diameter is the proper scaling parameter. It is not possible to discern from the data an effect of pipe diameter, which indicates that up to values of  $Z = 20$  the turbulent field is dominated by jet turbulence.